FRACTAL MODELING METHOD OF STOCHASTIC PROCESSES

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Abstract

This material represents modeling of generalized Brownian motion using wavelet-transformation, which is realized in MathCad software environment. Simulation is based on Hurst parameter. According to computer experiment, when Hurst parameter H=0.99 there is a persistent process, when H=0.1 there is anti-persistent process and when H=0.5 we get classical Brownian motion mapping, either, using or not using, wavelet-transformation, this proves that the proposed method is correct and Fractal modeling is suitable for researching of Stochastic processes.

Keywords: Brownian motion. Hurst parameter. Wavelet-transformation. Fractal modeling.

1. Introduction

It is known, that the research of complex systems functionality generally is based on its modeling [1]. Among different models Fractal modeling has its special place. Self-similarity is there exceptional characteristics [2]. Fractal modeling introduces new perspective field in computer researching. Modern researches are showed that telecommunications network traffic, which is an example of stochastic process, has fractal properties [3].

Using Fractal modeling in Stochastic Processes has its practical purpose, as much as, fractal like characteristics of traffic influences whole telecommunications network productivity. Models, that describe events of self-similar processes, use random fractals [2].

This work represents generalized modeling method of Brownian motion (Fractalic) that is based on using Hurst parameter and wavelet-transformation. MathCAD software is used for computer realization.

2. Basic part

Fractal process modeling first was realized by B. Mandelbrot [1] by introducing Brownian motion concept. Its defining parameter is Hurst index $0<H<1$. If $H=1/2$ there is a random process – Brownian classical motion [2]. But if $H\neq 1/2$ – then there is a generalized Brownian motion, of which notable characteristics is persistency or anti-persistency. When $H>1/2$ we get result that fits to keep the tendency of the process: if in the past, the values used to be increased it is also expected in the future and the opposite, the tendency of decrease in the past, means average decrease in the future. Such process is called persistent. H<1/2 case is characterized by anti-persistency. In this case, increase in the past randomly causes decrease in the future. All these processes are fractal-based.

It should be said, that generalized Brownian motion modeling algorithm, performed by Mandelbrot, is very complex, both as mathematically and as realization [2]. But using modern computer technologies and MathCAD software makes it easy to implement this problem [4].

In the first case, let's assign H=0,1. This case belongs to anti-persistent process creation, when the process is completely random without any forecasting.

$$
\mathrm{fBmScale}(noise, H, sd) := \left|
\begin{array}{l}
len \leftarrow \mathrm{rows}(noise) \\
numLev \leftarrow \log(len, 2) \\
v_0 \leftarrow 0 \\
\text{for } i \in numLev..1 \\
\quad v \leftarrow \mathrm{stack}\left[v, \mathrm{submatrix}\left(noise, \frac{len}{2^i}, \frac{len}{2^{i-1}} - 1, 0, 0\right) \cdot sd \cdot \left(2^i\right)^{H+\frac{1}{2}}\right] \\
v
\end{array}
\right.
$$

H=0,1 – Hurst parameter,

sd=0,5 – Standard deviation,

len=1024 – Time division, that shows the number of counters. Their numerical value is 2^{10}.

noise:=rnorm(len,0,1) – Vector of normally distributed values.

Function fBmScale(noise,H,sd) performs signal in wavelet-transformation area [5].

v := fBmScale(noise,H,sd)

v – Vector resulting from wavelet-transformation.

We prefer to get results of transformation in a time division and that's why we use wavelet back-transformation.

filter := symmlet(8)

w := idwt(v,log(rows(v),2),filter)

k := 0..len-1

k – Sequential number of counter, that is proportional of moment in time;

w – Amplitude of resulting signal for k value.

Figure 1 shows the result, that is appropriate for H=0,1 value of Hurst parameter and the change of process in time.

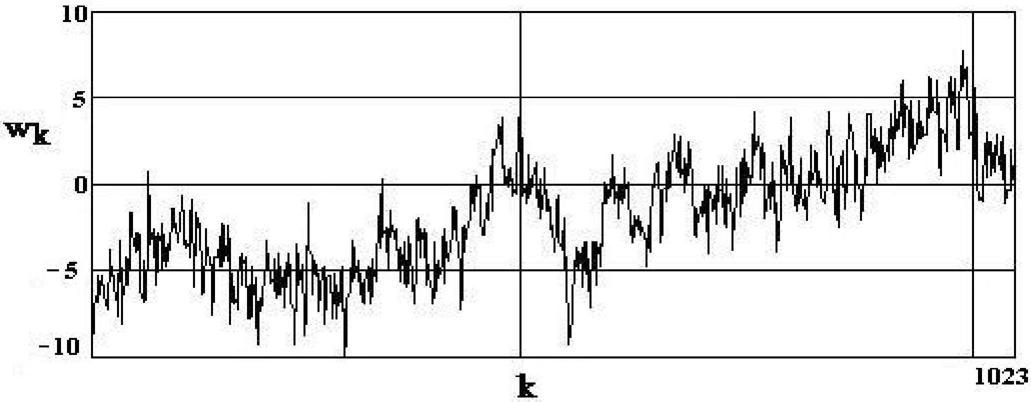

Fig.1 Time dependence chart received by wavelet back-transformation, when H=0.1

When H=1/2, generalized Brownian motion is transformed into classical Brownian motion. At this moment particle indentation is independent, instead of particle location in different moments. Particle indentation in one moment of time is not independent from indentation of this particle in another period of time.

The chart on Figure 2 shows the result, when Hurst parameter H=0.5 and wavelet transformation is used. To verify the correctness of the method we reflected Brownian motion without using wavelet transformation. The result is shown on picture.

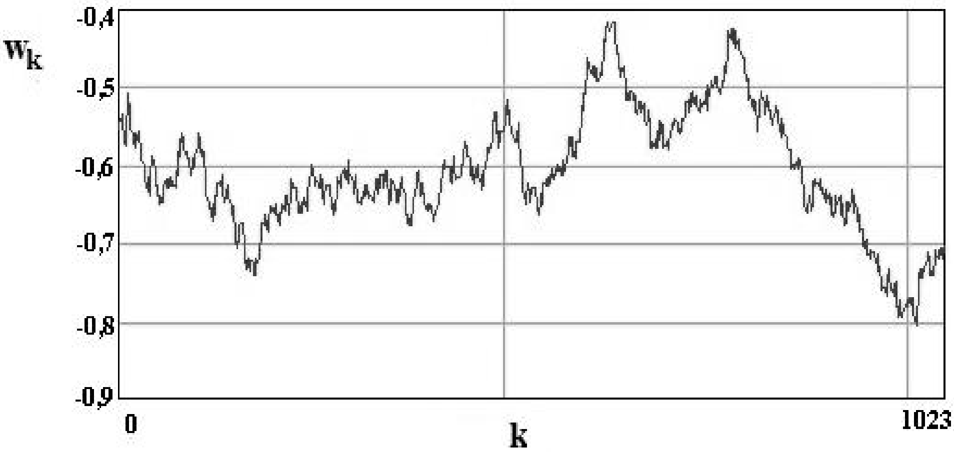

Fig. 2. The chart of classical Bwonian motion using wavelet transformation, when H=1/2

As the experiments showed the charts with (Fig.2) and without (Fig.3) wavelet transformation are almost identical, from here we can conclude that, it is better to do modeling of generalized Brownian motion with wavelet transformation.

Figure 4 shows the case when H=0.99. In this case the result approves that, when Hurst parameter is 0.99 the tendency of process is descending (when the time parameter is increasing). So the process becomes predictable, that means it has persistent properties.

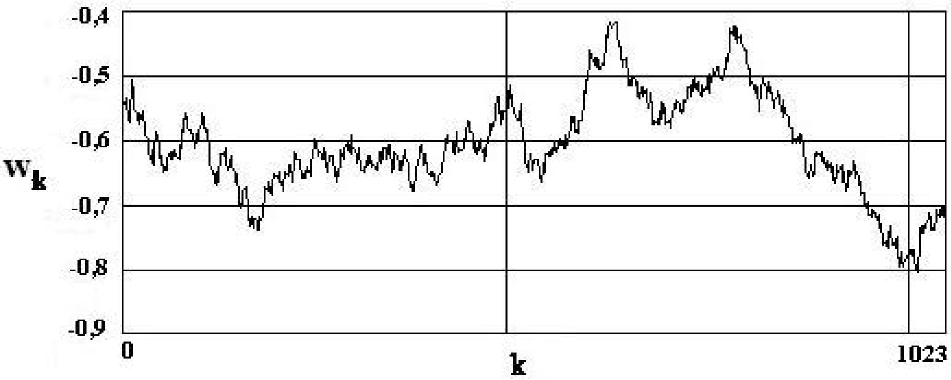

Fig.3. The chart of classical Bwonian motion without using wavelet transformation, when H=1/2.

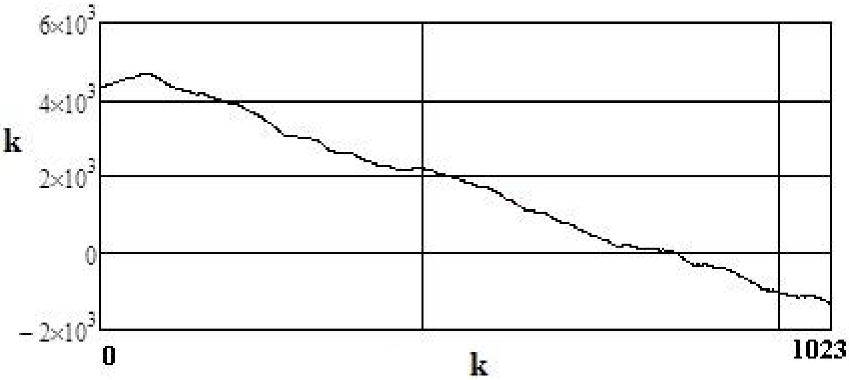

Fig.4. Modeling of generalized Brownian motion, when H=1/2.

3. Conclusion

According to the results we can conclude, that using wavelet transformation is more appropriate for modeling generalized Brownian motion and it has fractal properties (self-similarity). As much as network traffic (example of stochastic process) has fractal characteristics, suggested method of fractal modeling is suitable for researching stochastic processes.

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სტოქასტური პროცესების ფრაქტალური მოდელირების მეთოდი

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რეზიუმე

წარმოდგენილი მასალა ეხება განზოგადოებული ბროუნის მოძრაობის მოდელირებას ვეივლეტ-გარდაქმნის გამოყენებით, რაც რეალიზებულია MathCad პროგრამის გარემოში. მოდელირება ხდება ჰერსტის პარამეტრის საფუძველზე. კომპიუტერული ექსპერიმენტის შედეგად დადგინდა, რომ როცა ჰერსტის პარამეტრი H უდრის 0.99-ს, ვღებულობთ პერსისტენტულ პროცესს, როცა H=0.1-ს, ვღებულობთ ანტიპერსისტენტულ პროცესს, ხოლო როცა H=0.5-ს, ვღებულობთ კლასიკური ბროუნის მოძრაობის ასახვას, როგორც ვეივლეტ-გარდაქმნის გამოყენებით, ასევე ვეივლეტ-გარდაქმნის გამოყენების გარეშე, რამაც დაადასტურა შემოთავაზებული მეთოდის სისწორე და ფრაქტალური მოდელირების გამოყენებადობა სტოქასტური პროცესების კვლევებისათვის.

ФРАКТАЛЬНЫЙ МЕТОД МОДЕЛИРОВАНИЯ СТОХАСТИЧЕСКИХ ПРОЦЕССОВ

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Резюме

Этот материал представляет собой моделирование обобщенного броуновского движения с использованием вейвлет-преобразования, которая реализуется в программной среде MathCad. Моделирование основывается на параметре Херста. Согласно компьютерного эксперимента, когда параметр Херста H=0,99 получается персистентний процесс, когда H=0.1 - анти-персистентный а когда процесс H=0.5 получается классическое броуновское отображение движения, как при использовании вейвлет-преобразование, так и без его приенения. Это доказывает правильность использования предложенного метода, а фрактальное моделирование подходит для исследования стохастических процессов.